M 行 N 列の行列の変換::プログラム中のカウンタの説明

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{bmatrix}$$

1. 1 回目

   1,2 行目を 0 列から N 列まで変換 (カウンタは N-1 まで)

   1 行目の乗数は $\dfrac{a_{10}}{a_{00}} = \dfrac{1\text{行}0\text{列}}{0\text{行}0\text{列}} = \dfrac{1\text{行目の先頭}}{0\text{行目の対角要素}}$

   2 行目の乗数は $\dfrac{a_{20}}{a_{00}} = \dfrac{2\text{行}0\text{列}}{0\text{行}0\text{列}}$

$$\begin{bmatrix} \overset{a_{00}}{} & \overset{a_{01}}{} & \overset{a_{02}}{} & \overset{a_{03}}{} \\ a_{10} - \dfrac{a_{10}}{a_{00}}a_{00} & a_{11} - \dfrac{a_{10}}{a_{00}}a_{01} & a_{12} - \dfrac{a_{10}}{a_{00}}a_{02} & a_{13} - \dfrac{a_{10}}{a_{00}}a_{03} \\ a_{20} - \dfrac{a_{20}}{a_{00}}a_{00} & a_{21} - \dfrac{a_{20}}{a_{00}}a_{01} & a_{22} - \dfrac{a_{20}}{a_{00}}a_{02} & a_{23} - \dfrac{a_{20}}{a_{00}}a_{03} \end{bmatrix}$$

   a_{10} を置き換えるときは、別の変数名で保持する

2. 変換後

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ 0 & a_{11}{}' & a_{12}{}' & a_{13}{}' \\ 0 & a_{21}{}' & a_{22}{}' & a_{23}{}' \end{bmatrix}$$

3. 2 回目

   2 行目を 1 列から N 列まで変換 (カウンタは N-1 まで)

   2 行目の乗数は $\dfrac{a_{21}{}'}{a_{11}{}'} = \dfrac{2\text{行}1\text{列}}{1\text{行}1\text{列}} = \dfrac{2\text{行目のゼロでない先頭}}{1\text{行目の対角要素}}$

$$\begin{bmatrix} a_{00} & \overset{a_{01}}{} & \overset{a_{02}}{} & \overset{a_{03}}{} \\ 0 & a_{11}{}' & a_{12}{}' & a_{13}{}' \\ 0 & a_{21}{}' - \dfrac{a_{21}{}'}{a_{11}{}'}a_{11}{}' & a_{22}{}' - \dfrac{a_{21}{}'}{a_{11}{}'}a_{12}{}' & a_{23}{}' - \dfrac{a_{21}{}'}{a_{11}{}'}a_{13}{}' \end{bmatrix}$$

4. 2 回目変換終了

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ 0 & a_{11}{}' & a_{12}{}' & a_{13}{}' \\ 0 & 0 & a_{22}{}'' & a_{23}{}'' \end{bmatrix}$$

5. 後退代入

$$x_i = \frac{1}{a_{ii}{}'}\left\{ a_{i(M)}{}' - \sum_{j=i+1}^{M-1} a_{ij}{}' x_j \right\}, (i = M-1, M-2, \cdots, 2, 1, 0 = 2, 1, 0)$$

$$a_{22}{}''x_2 + (-)a_{23}{}'' = 0, x_2 = \frac{a_{23}{}''}{a_{22}{}''}$$
$$a_{11}{}'x_1 + a_{12}{}'x_2 + (-1)a_{13}{}' = 0, x_1 = \frac{1}{a_{11}{}'}\left(a_{13}{}' - a_{12}{}'x_2\right)$$
$$a_{00}{}'x_0 + a_{01}{}'x_1 + a_{02}{}'x_2 + (-1)a_{03} = 0, x_0 = \frac{1}{a_{00}{}'}\left(a_{03}{}' - \left(a_{01}{}'x_1 + a_{02}{}'x_2\right)\right)$$